Correspondence

Fourier Transforms and Directional Couplers*

The excellent review of Bolinder[1] on the applications of Fourier transforms in wave theory, particularly the section on "Coupling of Waves," suggested that the readers of these TRANSACTIONS might also be interested in some related work performed by this writer some time ago. A report[2] on this work contains a section on the use of Fourier integral and series methods for directional coupler design. In connection with directional couplers, this letter also can be considered as noting a further addition to the useful bibliography of Schwartz.[3] Although not widely distributed, the aforementioned report[2] has been available since late 1947 in the Document Room of the Research Laboratory of Electronics at the Massachusetts Institute of Technology as No. SPE-239. It should be emphasized that this letter is not intended to start a controversy as to who did what first, but rather to bring to the readers' attention a report of possible interest.

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1 E. F. Bolinder, "The relationship of physical applications of Fourier transforms in various fields of wave theory and circuitry," IRE TRANS., vol. MTT-5, pp. 153–158; April, 1957.

2 S. Sensiper, "Notes on Theory of Directional Couplers," Sperry Gyroscope Co., Inc., Great Neck, N. Y., Rep. No. 5224-1095; August 1, 1947.

3 R. F. Schwartz, "Bibliography on directional couplers," IRE TRANS., vol. MTT-2, pp. 58–63; July, 1954. See also, IRE TRANS., vol. MTT-3, pp. 42–43; April, 1955.

Analysis of the Variable-Ratio Microwave Power Divider*

It has been shown experimentally that a variable-ratio microwave power divider is easily assembled from two identical 3-db directional couplers and a phase changer.[1] The scattering matrix[2] of this structure is readily obtained from the flow graph[3] showing the interconnections between the couplers. The following shows that the whole structure is still a directional coupler, derives the conditions to optimize the power transfer, and finally, gives the phase angles of the scattering coefficients.

The type of lossless directional couplers that are considered here have the following scattering matrix

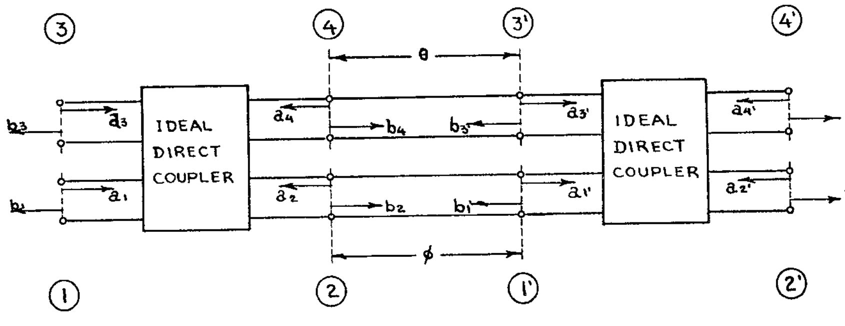

Fig. 1

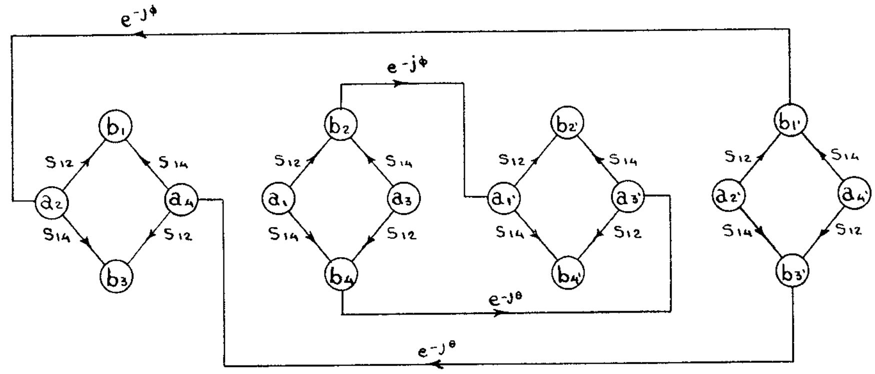

Fig. 2

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{14} & 0 \\ 0 & S_{14} & 0 & S_{12} \\ S_{14} & 0 & S_{12} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}, \quad (1)$$

where a_i and b_i are, respectively, the complex amplitudes of the incident and emergent waves referred to planes $i=1, 2, 3, 4$ and have been normalized in such a way that $a_ia_i^*$ and $b_ib_i^*$ are respectively proportional to the incident and emergent power at terminal i.

Fig. 1 shows the structure to be analyzed and Fig. 2 its flow graph. The whole structure can be specified by the following scattering matrix referred to planes 1, 2′, 3 and 4′

$$\begin{bmatrix} b_1 \\ b_{2'} \\ b_3 \\ b_{4'} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12'} & s_{13} & s_{14'} \\ s_{2'1} & s_{2'2'} & s_{2'3} & s_{2'4'} \\ s_{31} & s_{32'} & s_{33} & s_{34'} \\ s_{4'1} & s_{4'2'} & s_{4'3} & s_{4'4'} \end{bmatrix} \begin{bmatrix} a_1 \\ a_{2'} \\ a_3 \\ a_{4'} \end{bmatrix}. \quad (2)$$

From the flow graph the coefficients s_{ij} are found to be

$$s_{11} = s_{2'2'} = s_{33} = s_{4'4'} = 0$$
$$s_{13} = s_{31} = s_{2'4'} = s_{4'2'} = 0$$
$$s_{12'} = S_{12}^2e^{-j\phi} + S_{14}^2e^{-j\theta} = s_{2'1}$$
$$s_{14'} = S_{12}S_{14}(e^{-j\phi} + e^{-j\theta}) = s_{4'1} = s_{2'3} = s_{32'}$$
$$s_{34'} = S_{12}^2e^{-j\theta} + S_{14}^2e^{-j\phi} = s_{4'3}. \quad (3)$$

After substituting (3) in matrix (2), one obtains

$$\begin{bmatrix} b_1 \\ b_{2'} \\ b_3 \\ b_{4'} \end{bmatrix} = \begin{bmatrix} 0 & s_{12'} & 0 & s_{14'} \\ s_{12'} & 0 & s_{14'} & 0 \\ 0 & s_{14'} & 0 & s_{34'} \\ s_{14'} & 0 & s_{34'} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_{2'} \\ a_3 \\ a_{4'} \end{bmatrix}. \quad (4)$$

Since the analysis is based on the assumption of ideal lossless components, the scattering matrix (4) is unitary and of the same type as matrix (1). Thus the whole structure is still a directional coupler whose coefficients are functions of θ and ϕ. Let $\phi=\theta+\psi$, where ψ is variable at will (variable phase changer). Then,

$$s_{14'} = 2|S_{12}|\,|S_{14}| \cos\frac{\psi}{2} \cdot \exp j\left(\varphi_{12} + \varphi_{14} - \theta - \frac{\psi}{2}\right)$$

where φ_{12} = phase of S_{12}; φ_{14} = phase of S_{14}. The amplitude of $s_{14'}$ is then given by

$$|s_{14'}| = 2|S_{14}|\,[1 - |S_{14}|^2]^{1/2} \cos\frac{\psi}{2} \quad (5)$$

since $|S_{12}|^2+|S_{14}|^2=1$.

The optimization of $|s_{14'}|$ with respect to $|S_{14}|$ is obtained by making

$$\frac{d|s_{14'}|}{d|S_{14}|} = 0$$

for all possible values of ψ; *i.e.*;

$$\frac{d|s_{14'}|}{d|S_{14}|} = 2\cos\frac{\psi}{2}\sqrt{1 - |S_{14}|^2} \cdot \left[1 - \frac{|S_{14}|^2}{1 - |S_{14}|^2}\right] = 0.$$

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1 W. L. Teeter and K. R. Bushore, "A variable-ratio microwave power divider and multiplexer," IRE TRANS., vol. MTT-5, pp. 227–229; October, 1957.

2 C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., M.I.T. Rad. Lab. Ser., vol. 8, ch. 5; 1948.

3 S. J. Mason, "Feedback theory—some properties of signal flow graphs," PROC. IRE, vol. 41, pp. 1144–1156; September, 1953.